Amazon's hanging cable interview question.

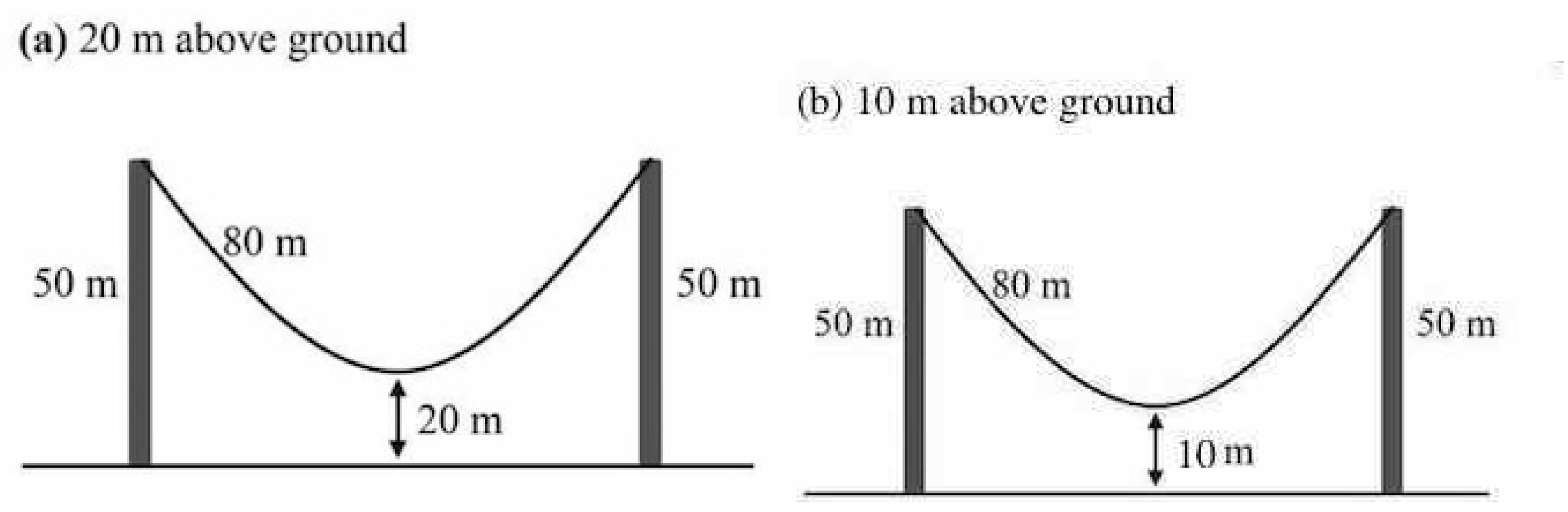
March 31, 2022

A cable of 80 metres is hanging from the top of two poles that are both 50 metres off the ground.

What is the distance between the two poles (to one decimal point) if the centre is

(a) 20 metres off the ground

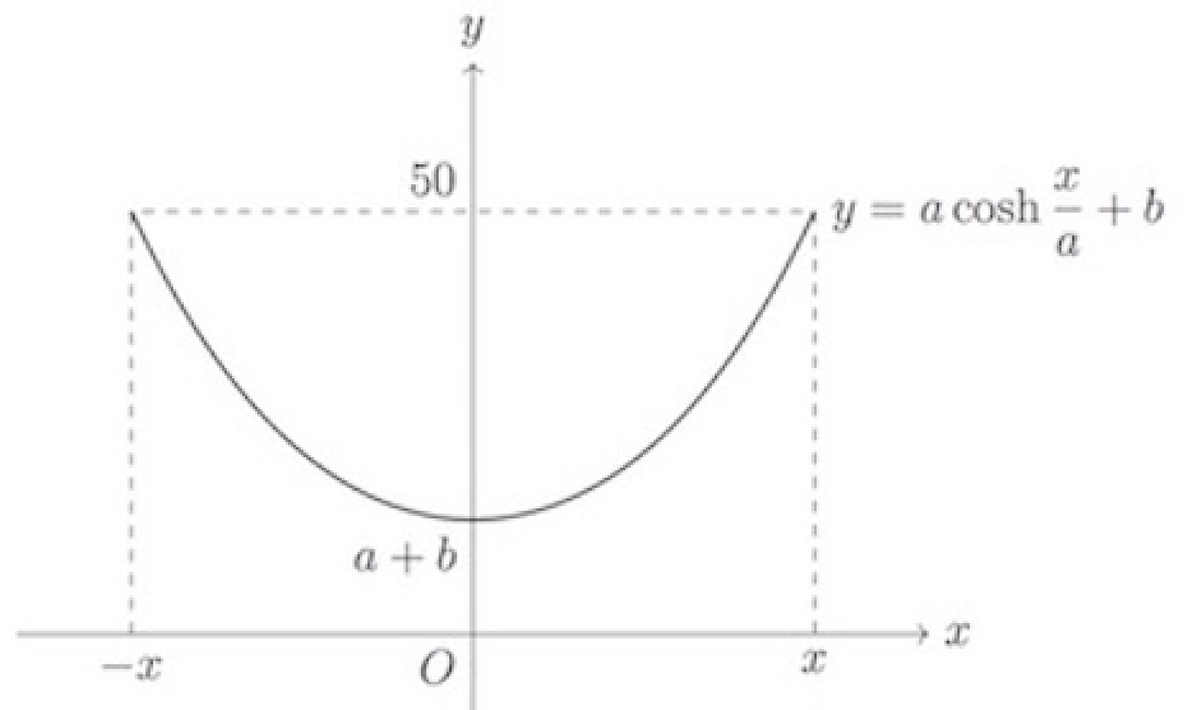
(b) 10 metres off the ground?



(a) The hanging cable is represented by a curve named "catenary".

The catenary is given by the hyperbolic function $y = a\cosh x + b$ (where a and b are constants) as the diagram:

The distance between two poles is $2x$.



We have equations:

$$y(0) = a + b = 20$$
$$y(x) = a\cosh x + b = 50$$

Then

$$a\cosh x + 20 - a = 50$$
$$a\cosh x = a + 30 \cdots (1)$$

Let $2L$ be the length of the cable.

Then $2L = 80$.

And a half of the length of the cable is given by the integration as

$$L = \int_0^x ds$$

where

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + (\frac{dy}{dx})^2}dx = \sqrt{1 + y'^2}dx$$

.

As $y = a\cosh\frac{x}{a} + b$,

$$y' = \sinh\frac{x}{a}$$

Then

$$ds = \sqrt{1 + \sinh^2\frac{x}{a}}dx = \sqrt{\cosh^2\frac{x}{a}}dx = \cosh\frac{x}{a}dx$$

Hence

$$L = \int_0^x ds = \int_0^x \cosh\frac{x}{a}dx = \left[a\sinh\frac{x}{a}\right]_0^x = a\sinh\frac{x}{a}$$

Then

$$a\sinh\frac{x}{a} = 40 \cdots (2)$$

Calculate $(1)^2$ - $(2)^2$,

$$a^2\cosh^2\frac{x}{a} - a^2\sinh^2\frac{x}{a} = (a+30)^2 - 40^2$$
$$a^2(\cosh^2\frac{x}{a} - \sinh^2\frac{x}{a}) = a^2 + 60a - 700$$
$$a^2 = a^2 + 60a - 700$$

Then

$$a = \frac{700}{60} = \frac{35}{3}$$

Substitute this in the term (2).

$$\frac{35}{3}\sinh\frac{3}{35}x = 40$$
$$\sinh\frac{3}{35}x = \frac{24}{7}$$
$$\frac{e^{\frac{3}{35}x} - e^{-\frac{3}{35}x}}{2} = \frac{24}{7}$$
$$7e^{\frac{3}{35}x} - 7e^{-\frac{3}{35}x} = 48$$

Let $X = e^{\frac{3}{35}x}$.

$$7X - 7X^{-1} = 48$$
$$7X^2 - 48X - 7 = 0$$
$$(X-7)(7X+1) = 0$$

Since $X > 0$,

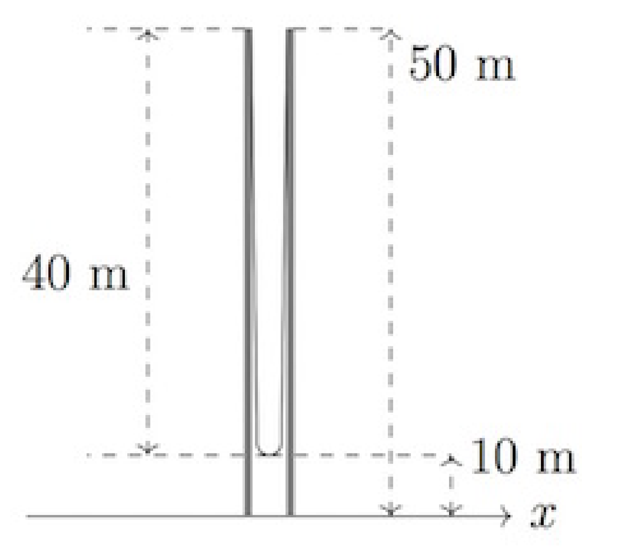
$$X = 7$$
$$e^{\frac{3}{35}x} = 7$$
$$\frac{3}{35}x = \log 7$$
$$x = \frac{35}{3}\log 7$$

Therefore the distance of two poles is

$$2x = \frac{70}{3}\log 7 = 45.4\ m$$

(b) If the centre is 10 metres off the ground,

The difference height between the top of the pole and the centre of the cable is 40 metres, which is the same length of a half of the cable.



Then the distance between two poles must be 0 metres.