Trigonometric Functions

Example 1

[1] Transform the following angles ... degree to radian and radian to degree.

(1) $495°$ (2) $-1200°$

(3) $\frac{5}{12}\pi$ (4) $-\frac{5}{4}\pi$

[2] Evaluate the following trigonometric functions.

(1) $\sin\frac{11}{3}\pi$ (2) $\cos(-\frac{3}{4}\pi)$

(3) $\tan\frac{7}{3}\pi$ (4) $\cos 7\pi$

[1] Transform the following angles ... degree to radian and radian to degree.

(1) $390°$ (2) $-120°$ (3) $1520°$

(4) $\frac{5}{4}\pi$ (5) $\frac{13}{3}\pi$ (6) $-\frac{11}{12}\pi$

[2] Evaluate the following trigonometric functions.

(1) $\sin\frac{5}{4}\pi$ (2) $\cos(-\frac{11}{6}\pi)$ (3) $\tan\frac{5}{3}\pi$

(4) $\sin(-\frac{9}{2}\pi)$ (5) $\cos 3\pi$ (6) $\tan(-\frac{7}{6}\pi)$

Example 2

Sketch the graph of following functions.

(1) $y = \sin(x - \frac{\pi}{3})$ (2) $y = 2\cos x$

(3) $y = \tan x$ (4) $y = -2\cos(\frac{\pi}{2}x + \frac{\pi}{4})$

[3] Sketch the graph of following functions.

(1) $y = -\sin x$ (2) $y = \frac{1}{2}\cos 2x$

(3) $y = \tan\frac{\pi}{2}$ (4) $y = -\sin(2x - \frac{\pi}{4})$

Example 3

Solve the following equations and inequalities, where $0 \leq x < 2\pi$.

(1) $2\sin^2 x - \cos x - 1 = 0$

(2) $4\sin^2 x + 2(1-\sqrt{3})\sin x - \sqrt{3} \geq 0$

(3) $\tan^2 x - 2\tan x + 1 = 0$

(4) $\tan^2 x - 1 > 0$

[4] Solve the following equations and inequalities, where $0 \leq x < 2\pi$.

(1) $2\cos^2 x + 3\sqrt{3}\sin x = 5 = 0$

(2) $\sqrt{2}\sin^2 x - \sin x \geq 0$

(3) $\sqrt{3}\tan^2 x - 2\tan x - \sqrt{3} < 0$

(4) $4\sin x\cos x - 2\sin x - 2\cos x + 1 < 0$

Example 4

Prove the following equalities.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

[5] Let $0 < \alpha < \dfrac{\pi}{2}, \quad \dfrac{\pi}{2} < \beta < \pi, \quad \cos\alpha = \dfrac{3}{5}, \quad \sin\beta = \dfrac{\sqrt{3}}{3}$. Evaluate the following expressions.

(1) $\cos(\alpha+\beta)$ (2) $\sin(\alpha-\beta)$ (3) $\tan(\alpha-\beta)$

Example 5

Prove the following equalities.

(1) $\sin 2\theta = 2\sin\theta\cos\theta$

(2) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

(3) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(4) $\sin^2 x = \dfrac{1-\cos 2x}{2}, \quad \cos^2 x = \dfrac{1+\cos 2x}{2}, \quad \tan^2 x = \dfrac{1-\cos 2x}{1+\cos 2x}$

[6] Evaluate the following expressions when $\dfrac{\pi}{2} < \alpha < \pi$ で $\tan\alpha = -\sqrt{7}$

(1) $\sin 2\alpha$

(2) $\cos 2\alpha$

(3) $\tan 2\alpha$

(4) $\sin 3\alpha$

[7] Evaluate $\sin\dfrac{\alpha}{2}$, $\cos\dfrac{\alpha}{2}$, $\tan\dfrac{\alpha}{2}$, when $0 < \alpha < \dfrac{\pi}{2}$, $\quad \cos\alpha = \dfrac{\sqrt{2}}{4}$.

Example 6

Prove the following expressions.

(1) $a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}\sin(\theta+\alpha)$, where $\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}, \quad \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

(2) $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

[8] Solve the ofllowing equations.

(1) $\sin 2x + \sin 5x = 0$

(2) $\cos 2x - 3\cos x + 2 = 0 \quad (0 \leq x < 2\pi)$

(3) $\cos x + \cos(x - \dfrac{\pi}{3}) = 0 \qquad (0 \leq x < 2\pi)$

(4) $\sin 2x + \sin 3x - \sin 7x = 0 \qquad (-\dfrac{\pi}{2} < x \leq \dfrac{\pi}{2})$

(5) $\sin x + \sqrt{3}\cos x = -1 \qquad (0 \leq x < 2\pi)$

(6) $\sqrt{3}\sin 2x + 2\cos^2 x - 1 = 0 \qquad (0 \leq x < 2\pi)$

[9] Solve the following inequalities.

(1) $\sin 5x + \sin 3x > 0 \qquad (0 \leq x < \pi)$

(2) $\cos 2x + 3\sin x \geq 1 \qquad (0 \leq x < 2\pi)$

(3) $\cos 8x + \cos 3x + \cos 2x < 0 \qquad (0 \leq x < \dfrac{\pi}{2})$

(4) $\sin x + \sqrt{3}\cos x < 1 \qquad (0 \leq x < 2\pi)$

Example 7

Find the maxima and minima of the following functions.

(1) $y = sinx + \cos x \quad (0 \leq x \leq \pi)$.

(2) $y = 2\sin^2 x + \cos x - 1$

[10] Find the maxima and minima of the following functions.

(1) $y = 2\cos^2 x + \sin x - 1$

(2) $y = -\sqrt{2}\sin x + \sqrt{6}\cos x \qquad (0 \leq x < \pi)$

[11] Find the maxima and minima of $y = \sin x \cos x - \sqrt{3}\sin^2 x + \sqrt{3} \qquad (0 \leq x \leq \dfrac{\pi}{2})$.

Exercises

[1] When $\sin^3\theta + \cos^3\theta = \dfrac{13}{27}$ $(\dfrac{\pi}{2} < \theta < \pi)$, evaluate $\sin\theta$ and $\cos\theta$.

[2] (1) Let $t = \sin\theta + \cos\theta$ write down $\sin\theta\cos\theta$ with t.

(2) When $0 \leq \theta \leq \pi$, find the range of $t = \sin\theta + \cos\theta$.

(3) Check the number of real roots of the equation

$$2\sin\theta\cos\theta - 2(\sin\theta + \cos\theta) - k = 0$$

when $0 \leq \theta \leq \pi$.

[3] (1) Write down $\sin 4x$ with $\sin x$ and $\cos x$.

(2) Find the range of real number a when the equation $\sin 4x = a\sin x$ has 3 different roots in $0 \leq x \leq \dfrac{\pi}{2}$.

[4] Given a quadratic function $f(x) = x^2 + (\cos\theta)x + \sin\theta - 1$ with $0 \leq \theta \leq 2\pi$.

(1) Find the vertex of $y = f(x)$.

(2) Evaluate the number of intersection between $y = f(x)$ and x-axis.

(3) Find the range of θ that every roots of $f(x) = 0$ are in the interval $-1 < x < 1$.

[5] Let $f(\theta) = a\sin^2\theta + \sqrt{3}(b-a)\sin\theta\cos\theta + b\cos^2\theta$, and let a, b be real numbers as $a < b$.

(1) Write down $f(\theta)$ with a, b, $\sin 2\theta$ and $\cos 2\theta$.

(2) Evaluate the maxima and minima of $f(\theta)$ with a and b, when $0 \leq \theta \leq \frac{\pi}{2}$.

[6] Given two functions of t as

$$\begin{cases} f(t) = t\cos\theta \\ g(t) = t\sin\theta - \frac{1}{2}t^2 \end{cases} \quad \left(0 < \theta < \frac{\pi}{2}\right)$$

(1) Write down the range of t with θ, when $g(t) \geq 0$.

(2) When t changes in the range of (1), find the maximum L of $f(t)$, and the minimum H of $g(t)$; using θ.

(3) Considering L as a function of θ, where $0 < \theta < \frac{\pi}{2}$, find the value of θ when L becomes the maximum.

[7] For finding the value of $\cos\left(\frac{2\pi}{5}\right)$, let

$$\cos\left(\frac{2\pi}{5}\right) = t$$

(1) Write down $\cos\left(\frac{\pi}{10}\right)$ with t.

(2) Find a polynomial $P(x)$, which satisfies

$$\cos(5\theta) = P(\cos\theta)$$

for every θ.

(3) Evaluate t.