Exponential and logarithm

Exemple 1

Express the following expressions as exponential form as a^r.

(1) $a^{\frac{1}{2}} \times a^{\frac{1}{3}} \div a^{\frac{1}{6}}$

(2) $a^{-\frac{1}{2}} b^{\frac{1}{3}} \times a^{\frac{3}{4}} b^{-\frac{5}{6}}$

(3) $\sqrt{\dfrac{xz^3}{y^2}} \div \sqrt[4]{\dfrac{x^2y}{z^3}}$

(4) $\sqrt[4]{\sqrt{a} \times \dfrac{a}{\sqrt[3]{a}}}$

[1] Express the following expressions as exponential form as a^r.

(1) $(a^{\frac{1}{2}} \cdot a^{-\frac{1}{2}})^1 5$

(2) $\{(\dfrac{125}{64})^{\frac{1}{4}}\}^{-\frac{2}{3}}$

(3) $a \times (a^{-2}b^{-3})^{-2} \div (ab^{-1})^3$

(4) $\{(\dfrac{16}{125})^{-\frac{3}{4}}\}^{-\frac{9}{2}}$

[2] Evaluate $\dfrac{3^{3x} + 3^{-3x}}{3^x + 3^{-x}}$, when $3^{2x} = 5$.

Example 2

Sketch graphs of the following functions.

(1) $y = 2^x$

(2) $y = (\frac{1}{2})^x$

(3) $y = 2^{x-1}$

(4) $y = -(\frac{1}{2})^x$

[3] Solve the following equations and inequalities.

(1) $5^x = 625$

(2) $4^x = 8$

(3) $(\sqrt{2})^x = 32 \cdot 2^{-2x}$

(4) $2^x \geq 256$

(5) $(\frac{2}{3})^x \leq \frac{9}{4}$

(6) $2^{2x-3} \cdot 2^x + 2 > 0$

Example 3

Evaluate the following expiessions.

(1) $\log_2 8$

(2) $\log_2 \sqrt[3]{2}$

(3) $\log_3 \dfrac{1}{81}$

(4) $\log_{10} \dfrac{1}{1000}$

[4] Evaluate the following expressions.

(1) $\log_3 \sqrt[5]{9}$

(2) $\log_{10} 100\sqrt{10}$

(3) x when $\log_x 243 = 5$

(4) x when $\log_x \sqrt[4]{3} = \dfrac{1}{2}$

Example 4

Simplify the following expressions.

(1) $(\log_2 \dfrac{9}{4})^2 - (\log_2 9)^2 + 2\log_2 81$

(2) $(\log_4 3) \cdot (\log_9 25) \cdot (\log_5 8)$

[5] Simplify the following expressions.

(1) $(\log_{10} 2)^3 + (\log_{10} 5)^3 + (\log_{10} 5) \cdot (\log_{10} 8)$

(2) $\dfrac{1}{2}\log_2 10 + \log_4 14 - 3\log_8 \sqrt{35}$

(3) $(\log_2 6) \cdot (\log_3 6) - (\log_2 3 + \log_3 2)$

(4) $(\log_2 3 + \log_4 9)(\log_3 4 + \log_9 2)$

Example 5

Solve the following equations and inequalities.

(1) $\log_{10} x + \log_{10}(x-1) = \log_{10} 2 + 1$

(2) $(\log_2 x)^2 - \log_2 x^2 + 2 = 0$

(3) $\log_a(2x-1) \geq \log_a(x+1) \quad (a > 0, a \neq 1)$

(4) $(\log_{10} 4x)(\log_{10} 8x) \leq 12(\log_{10} 2)^2$

[6] Solve the following equations and inequalities.

(1) $\log_4(4+x-x^2) = \dfrac{1}{2} + \log_2(1-x)$

(2) $(\log_2)^2 - 2\log_2 x - 3 \geq 0$

(3) $(\log_3 x)(\log_3 \dfrac{x}{9}) = 8$

(4) $3 \cdot 2^{2x-2} + 2^{x-1} - 2^{-2} \leq 0$

Example 6

(1) Suppose that $0 \leq x \leq 2$. Find the maximum and minimum of $y = 2^{1-2x} - 2^{1-x} + 1$ and find the values of x at these points.

(2) Find the minimum of $y = (\log_2 x)^2 + \log_4 \dfrac{x^8}{4}$ and the value of x at this point.

[7] Given $f(x) = 2^{2x} + \dfrac{1}{2^{2x}} - 3\sqrt{2}(2^x + \dfrac{1}{2^x}) + 4$,

(1) Putting $2^x + \dfrac{1}{2^x} = t$, express $f(x)$ with t.

(2) Find the minimum of $f(x)$ and the value of x at this point.

[8] Find the minimum of $2x + 3y$, when $\log_3 x + \log_3 y = 5$

Example 7

Let $\log_{10} 2 = 0.3010,\ \log_{10} 3 = 9.4771$.

(1) Find the digits of number 15^{15}.

(2) How many zeros does continue when you express $(\frac{5}{9})^{100}$ as a decimal number ?

(3) Show that $8^m > 9^n$,when $100m > 106n$, where $m,\ n$ are positive integers.

[9] Let $\log_{10} 2 = 0.3010,\ \log_{10} 3 = 9.4771,\ \log_{10} 7 = 0.84510$.

(1) Find the range of $\log_{10} x$ when $60000 \leq x \leq 70000$.

(2) Find the digits of number 3^{2011}.

(3) Find the highest digit's number of 3^{2011}

(4) Find the lowest digit's number of 3^{2011}

[10] Let $\log_{10} 2 = 0.3010,\ \log_{10} 3 = 9.4771$.

When the light passes a glass, it's quantity loses $\frac{1}{10}$. Using such glasses, we want that the quantity of light is less than $\frac{1}{5}$ of the initial quantity. Find the minimum number of glasses, which we need. Let $\log_{10} 2 = 0.3010,\ \log_{10} 3 = 9.4771$

Exercises

[1] Evaluate $\log_2(\sqrt{5+\sqrt{24}} - \sqrt{5-\sqrt{24}})$

[2] Evaluate $\log_5 25 + \log_5 75 - \log_5 3 + \dfrac{\log_5 27}{\log_5 9}$.

[3] Solve the inequality $\log_2 x - 3\log_x 2 > 2$.

[4] Find the maximum and minimum of $y = 2\log_5 x + (\log_5 x)^2$, when $1 \leq x \leq 5$

[5] Solve the following system of equations

$$\begin{cases} \log_2 xy = \log_2 x \cdot \log_2 y \\ \log_2 \dfrac{y}{x} = \dfrac{3}{2} \end{cases}$$

[6] Let a be a constant number as $a > \frac{1}{2}$. When $1 \leq x \leq 2a$, find the maximum and minimum of

$$y = (\log_2 \frac{x}{a})(\log_2 \frac{x^2}{a^2})$$

[7] Given a sequence $\{a_n\}$ as

$$a_1 = 1, \quad a_{n+1} = 2a_n^2 \quad (n = 1,\ 2,\ 3,\ \cdots)$$

(1) Express a_n with n.

(2) How many positive integers n satisfying $a_n < 10^{60}$.

For question (2), you use $\log_{10} 2 = 0.3010$.