Coordinate geometry

Example 1

[1] Calculate the distance between following two points.

(1) $A(1,2),\ B(-5,2)$ (2) $A(-3,-1),\ B(-3,7)$

[2] (1) Find the point which divides internally in the ratio $1:2$ of the segment AB, where $A(1,1),\ B(-3,2)$,

(2) Find the point which divides externally in the ratio $1:2$ of the segment AB, where $A(1,1),\ B(-3,2)$,

(3) Find the midpoint of the segment AB, where $A(-1,5),\ B(-3,2)$.

[1] Calculate the distance between following two points..

(1) $A(1,-2),\ B(5,2)$ (2) $A(3,1),\ B(-3,-7)$

[2] (1) Find the point which divides internally in the ratio $2:3$ of the segment AB, where $A(6,-1),\ B(-3,-2)$,

(2) Find the point which divides externally in the ratio $3:1$ of the segment AB, where $A(2,1),\ B(3,2)$,

(3) Find the midpoint of the segment AB, where $A(4,5),\ B(-3,2)$.

[3] Find the centre of gravity of the triangle ABC , where $A(1,-1),\ B(3,2),\ C(-1,5)$.

Example 2

Find the equation of the line, which satisfies the following condition.

(1) Line passing through the point $A(1, 3)$ and it's gradient is -3.

(2) Line passing through two points $A(-2, 3),\ B(3, -1)$.

(3) Line ssing through two points $A(-5, 1),\ B(-5, \sqrt{3})$.

(4) Line ssing through the point $A(-3, -7)$ and parallel to the line $y = -2x + 11$.

(5) Line passing through the point $A(-3, -7)$ and perpendicular to the line $y = -2x + 11$.

[4] Find the equation of the line, which satisfies the following condition.

(1) Line passing through $A(-5, 2)$ and it's gradient is $\frac{1}{2}$.

(2) Line passing through two points $A(2, -3),\ B(-3, -1)$.

(3) Line passing through two points $A(\sqrt{5}, 1),\ B(\sqrt{5}, \sqrt{3})$.

(4) Line passing through $A(-3, -7)$ and parallel to the line $x = -1$.

(5) Line passing through $A(5, 1)$ and perpendicular to the line $y = \frac{1}{3}x + 1$.

Example 3

Find the equation of the circle, which satisfies the following condition.

(1) Circle whose centre is $(1, -2)$ and its radius is 3.

(2) Circle whose edges of the diameter are$(1, -1)$, $(3, 3)$.

(3) Circle passing through three points $(1, 0)$, $(3, -2)$, $(-1, 5)$.

(4) Circle passing through $(2, 3)$ and tangent to the both x-axis and y-axis.

[5] Find the equation of the circle, which satisfies the following condition.

(1) Circle whose centre is $(1, 1)$ and its radius is 2.

(2) Circle whose edges of the diameter are$(1, 0)$, $(-3, 3)$.

(3) Circle passing through three points $(2, 2)$, $(-3, -2)$, $(1, 5)$.

(4) Circle passing through $(-2, 4)$ and tangent to the both x-axis and y-axis.

(5) Circle passing through $(1, 1)$, $(-3, 5)$ and tangent to the both x-axis.

Example 4

(1) Find the distance between the point $(0, 0)$ and the line $ax + by + c = 0$.

(2) Find the distance between the point (x_0, y_0) and the line $ax + by + c = 0$.

[6] Find the distance between the following point and line.

(1) $(1, 1)$, $3x - y = 3$

(2) $(-2, 4)$, $y = -x + 5$

(3) $(0, 0)$, $y = x + 1$

(4) $(-3, 2)$, $y = -1$

[7] Given the triangle ABC, where $A(1, -1)$, $B(3, 2)$, $C(-1, 5)$.

(1) Find the equation of the line passing through A, B.

(2) Find the distance between the point C and the line AB.

(3) Estimate the wire od the triangle ABC.

Example 5

(1) Find the equation of the tangent of the circle $x^2 + y^2 = r^2$ at the point (x_0, y_0).

(2) Find the equation of the tangent of the circle $(x-a)^2 + (y-b)^2 = r^2$ at the point (x_0, y_0).

(3) Find the equation of the line, which passes on the point $A(2, -5)$ and tangent to the circle$(x+1)^2 + (y-3)^2 = 4$.

[8] Find the equation of the tangent od the following circle at the given point.

(1) $x^2 + y^2 = 4, \quad (-1, \sqrt{3})$

(2) $x^2 + y^2 - 2x - y = 0, \quad (0, 0)$

[9] Find the equation of the tangent of the circle $(x+1)^2 + (y-1)^2 = 4$, and passes on the point $(1, 7)$.

Example 6

[1] Check the relation (intersect with two different points, tangent or no common points) between the following circle and line.

(1) $x^2 + y^2 = 5, \quad y = -x + \sqrt{10}$

(2) $(x-1)^2 + (y-1)^2 = 4, \quad x - y = 1$

(3) $(x+2)^2 + (y-1)^2 = 3, \quad y = 3x = 10$

[2] Let $A,\ B$ the two points intersecting of $x^2 + y^2 - 4x + 2y - 5 = 0$ and $y = -x + 1$. Find the length of the segment AB.

[10] Check the relation (intersect with two different points, tangent or no common points) between the following circle and line.

(1) $x^2 + y^2 = 4, \quad y = -x + 5$

(2) $(x+1)^2 + (y-1)^2 = 5, \quad y = 2x + 1$

(3) $(x-2)^2 + (y+1)^2 = 2, \quad y = x - 1$

[11] Find the distance between the two intersecting points of $(x+3)^2 + (y-1)^2 = 5$ and $y = 2x - 6$.

Example 7

[1] Check the relation (intersect with two different points, tangent or no common points) between the following circles

(1) $x^2 + y^2 = 2, \quad (x-3)^2 + (y+4)^2 = 9$

(2) $(x-1)^2 + (y-1)^2 = 4, \quad x^2 + (y-1)^2 = 2$

(3) $(x+2)^2 + (y-1)^2 = 3, \quad (x-1)^2 + (y+3)^2 = 2$

[2] Find the equation of the circle which passes the two intersecting points of $(x-1)^2 + (y-1)^2 = 4$ and $(x-2)^2 + y^2 = 1$, and the origin $(0,0)$.

[12] Check the relation (intersect with two different points, tangent or no common points) between the following circles

(1) $x^2 + y^2 = 1, \quad (x+3)^2 + (y+4)^2 = 16$

(2) $(x+1)^2 + (y-2)^2 = 4, \quad x^2 + (y+1)^2 = 2$

(3) $(x+2)^2 + (y-5)^2 = 3, \quad (x-1)^2 + (y-3)^2 = 1$

[13] find the equation of the line passing through the two intersecting points of $(x-1)^2 + (y-1)^2 = 4$ and $(x-2)^2 + y^2 = 1$.

Example 8

[1] Draw the region defined by the following inequalities.

(1) $y > x + 1$ and $y < -x + 1$

(2) $(x-1)^2 + (y-1)^2 \leqq 4$

(3) $(x+2)^2 + (y-1)^2 \geqq 4$ or $(x-1)^2 + (y+3)^2 \leqq 9$

[2] Let $P(x, y)$ be a point moving in the region defined by $y - 2x + 4 \geqq 0,\ y + 3x - 5 \geqq 0,\ 2y + x - 12 \leqq 0$.

(1) Find the maximum and minimum of $x + y$.

(2) Find the maximum and minimum of $x^2 + y$.

[14] Draw the region defined by the following inequalities.

(1) $(x + y - 1)(2x + y) \leqq 0$

(2) $(x-1)^2 + (y-1)^2 \geqq 4$

(3) $(x+2)^2 + (y-1)^2 \geqq 4$ and $y \leqq -x + 1$

[15] When a point $P(x, y)$ moves in the region drfined by $x - 2 \leqq y \leqq -x^2$, find a maximum and minimum of $x + y$.

Example 9

[1] Find the equation of the trace of the point P, which satisfies the following condition.

(1) Let $A(1,1)$ and $B(-1,-1)$. $PA : PB = 2 : 1$.

(2) P is the same distance from the two lines $y = -x + 2$ and $y = 2x - 1$.

[2] Let Q be the point moving on the circle $x^2 + y^2 = 4$. Find the equation of the trace of the point P, which divides internally the segment attaching Q and $A(7,0)$ in the ratio $3 : 1$.

[16] Find the equation of the trace of the point P, which satisfies the following condition..

(1) P is the same distance from $A(1,1)$ and $B(-1,-1)$.

(2) P is the same distance from $A(0,1)$ and the line $y = -1$.

[17] Find the equation of trace of the point $Q(p+q, pq)$, when the point $P(p,q)$ is moving on the line $y = 2x$.

Exercises

[1] Find the equation of the line which passes through (2, 0) and the intersection go the following two lines.

$$3x - 2y - 4 = 0, \quad 4x + 3y - 10 = 0$$

[2] Let $a > 0$ be a real number, and the line $\ell : y = \frac{4}{3}x$ be tangent to the circle $C : (x-a)^2 + y^2 = 9^2$.

(1) Find the value of a.

(2) Let C_1 be a circle different from C, whose centre id on the x-axis and tangent both to ℓ and C. Find the centre and radius of C_1.

[3] Let a, b be real numbers and suppose that $y = x^3 - 3ax^2 - 3bx$ has its extrema at $x = p$and $x = q$.

(1) Draw the region of points (a, b) satisfying the condition $-1 \leqq p \leqq 0$ and $1 \leqq q \leqq 2$.

(2) When the point (a, b) moves in the region (1), find a maximum and minimum of $a + b$.

[4] Given $C : y = k - x^2$ を C, where $k > \frac{1}{2}$.

Let $P\left(t, k - t^2\right)$ be a moving point on C and $t \geqq 0$. Let P_0 be the point P, when the length of OP is minimum.

(1) Find P_0.

(2) Prove that the line OP_0 is perpendicular to the tangent of C at P_0.

(3) When the gradient of OP_0 is 1, find k.

(4) When the gradient of OP_0 is 1, find the radius of the circle, which is in the first quadrant and tangent to x-axis, y-axis and C.

[5] Let D be the region defined by $|x+2y|+|2x-y| \leqq 1$.

(1) Draw D.

(2) Find the maximum and minimum of $x+y$ on$D$.

(3) Find the maximum and minimum of $|x|+|y|$ on$D$.

[6] Let C be the circle whose centre is the origin O and its radius is 1, and suppose $P(p,q)$ satisfies the condition $p^2+q^2>1$. Let $T(s,t)$ be the tangent point, where the tangent line of C passes through the point P. Let D be the circle, whose centre is P and passes through T and suppose D passing through the point $A(a,\ 0)$.

(1) Prove that $(a-p)^2 = p^2-1$.

(2) When $0<a<1$ show that $p>1$ and write down a by using p.

[7] Let C be the circle whose centre is $P(0,\ 1)$ and its radios is 1. Let $a$be a real number satisfying $0<a<1$, and let $Q,\ R$ be the intersection of $y=a(x+1)$ and C.

(1) Find the wire $S(a)$ of the triangle $\triangle$PQR.

(2) Find a, when $S(a)$ is the maximum.

[8] Given three lines

$$\ell : x+y=0, \quad \ell_1 : ax+y=2a+2, \quad \ell_2 : bx+y=2b+2$$

where $a,\ b$ are real numbers.

(1) The line ℓ_1 always passes through the point P no matter what number a. Find P.

(2) Find the condition of $a,\ b$, when the three lines $\ell,\ \ell_1,\ \ell_2$ make a triangle.

(3) Suppose $a,\ b$ satisfy (2). Find the range of $a,\ b$, when $(1,1)$ is in the triangle (2), and draw the region of (a,b).