AO Entrance Exams for Tokyo Institute Technology University 2011

[1] Find every natural number n, which satisfies the condition : $n!$ is a multiple of n^2.

[2] Find the range of

$$\frac{a^2+b^2+c^2}{ab+bc+ca}$$

where $a,\ b,\ c$ are changing as the length of three sides of triangle.

[3] Find every natural number n, such that there exits n-th degree polynomial $f(x)$ satisfying the condition : $f(x^2+1)=f(x)^2+1$.

[4] Given the regular enneagon (9 vertices), inscribed the circle with radius 1. Find five regular n-gons, whose vertices are on the sides of the regular enneagon. And each n, find the length of the side of these regular n-gons.

Entrance Exams for Kyoto University 2011

[1] (1) In a box there are 9 cards, on which are written the number from 1 to 9 each, and on every card the different number is written. From this box we pick two cards at the same time and let X be the smallest number between two cards. We put back these cards into the box, and pick two cards at the same time again, and let Y be the smallest number between two cards. Find the probability of $X = Y$.

(2) Evaluate $\int_0^{\frac{1}{2}} (x+1)\sqrt{1-2x^2}dx$

[2] Let $a,\ b,\ c$ be real numbers, and let T be a linear transformation presented by $\begin{pmatrix} a & 1 \\ b & c \end{pmatrix}$, which satisfies foliowing two conditions :

(i) T transforms $(1, 2)$ to $(1, 2)$

(ii) When T transforms $(1, 0)$ to a point A, and $(0, 1)$ to a point B, then the area of the triangle $\triangle$OAB is $\frac{1}{2}$

Find the values of $a,\ b,\ c$.

[3] Find the area surrounded by two following curves :

$$y = x, \quad y = |\frac{3}{4}x^2 - 3| - 2$$

[4] Prove the following inequality

$$(1-a_1)(1-a_2)\cdots(1-a_n) > 1 - (a_1 + \frac{a-2}{2} + \cdots + \frac{a_n}{2^{n-1}})$$

when $\frac{1}{2} < a_j < 1 \quad (j = 1,\ 2,\ \cdots,\ n)$ and $n \geq 2$ is a integer.

[5] Prove that the sphere whose origin is $O(0, 0, 0)$ and radius is $\sqrt{6}$ and the plane passing three points $(4, 0, 0),\ (0, 4, 0),\ (0, 0, 4)$ intersect each other, and find the range of the value xys when (x, y, z) moves on this intersection.

[6] Prove that there exist a sphere, which passes every 4 vertices $A,\ B,\ C,\ D$ of a tetrahedron.